monopulse radar, designed to eliminate angle error caused by amplitude noise, was subject to another source of aimpoint wander.[5]

The terms "glint" and "wander" are associated mainly with target angle noise, but in some usage it could include both angle noise and error caused by target amplitude noise.[3] However, the authors provide lengthy discussion of glint and aimpoint wander as a phenomenon resulting from amplitude noise; yet, they were not aware of angle noise, the major and most difficult component to reduce. Amplitude noise contribution to angle error can be eliminated by use of monopulse tracking or homing, but angle noise is a basic cause of angle error in any tracking or homing system. It can be reduced only by frequency agility or very high resolution techniques which are costly to implement.

Angle noise is a phase-front distortion phenomenon which, similar to amplitude noise, results from the interference between the echoes from the parts of a complex target. However, angle noise is directly dependent on the angular location of each reflecting element of the target having a magnitude proportional to the radius-of-gyration of the distribution of the reflecting areas of the target. Most radar devices find target direction by measuring the angle of the phase-front of a target echo and determine target direction by assuming the target is normal to the phase-front. Although the phase-front of the echo from a point source is essentially flat (at typical target ranges) and normal to the direction of the source, it can be warped by the interference phenomenon when the source is complex. The radar may observe a relatively flat portion of the warped phase-front, but the warping causes it to be tilted so that a normal to the phase-front no longer goes through the target and gives a false measure of the direction of the target.

A simple experiment readily separates the two sources of angle error in a tracking or homing system and demonstrates their characteristics. First, a beacon-type source of radar signal (which could be a pulsed rf signal generator synchronized to the radar prf) may be amplitude modulated at a low frequency (near the conical scan-rate) and connected to a single radiating element. This will cause errors in a conical scan radar when the modulation frequency (or harmonics) falls within the frequency range of the scan frequency plus or minus the servo bandwidth. The angle errors can be severe depending upon the modulation index. However, a monopulse radar will track the source with no error since the point source causes no angle noise or phase-front distortion.

Angle noise can be similarly demonstrated by dividing the rf beacon source between two radiating elements with adjustable relative phase and relative amplitude. With a small separation between the sources (compared to radar beamwidth), they can be adjusted in relative amplitude and phase to cause very large errors in either a conical scan or monopulse radar. For example, with 180 deg relative phase (as observed at the radar) and a relative amplitude of 0.9, the apparent location of the source will appear approximately 9.5 times the angular span of the two sources from their midpoint. The direction of error is to the side of the larger of the two sources. This angle noise phenomenon demonstration is performed with no modulation (other than radar pulse modulation if used), thus eliminating amplitude noise. With a fixed signal level and fixed values of relative amplitude and phase, the tracking or homing devices will look continuously in the erroneous direction with the direction error as described above. This same phenomenon has even been demonstrated in sonar systems.

The authors draw a false conclusion that the "wander" cannot exceed the span of the target. There is no logic to this conclusion if the wander is caused by amplitude noise because even a point source, modulated appropriately as described above, can cause large errors. Also, I have observed conical scan tracking of a propeller driven aircraft where the scan rate coincided with a propeller modulation frequency harmonic. The angle errors were spectacular, greatly exceeding the target span.

There might be some logic to the expectation that angle noise is limited to the extent of the span of the target. However, theory and measurement of radar tracking of aircraft demonstrate that the apparent location of a target (even exclusive of multipath) can fall far outside the target span. Also, multipath readily demonstrates this phenomenon. For a target composed of an aircraft and its reflected (multipath) image, the apparent angular location can be in error by many times the target span (target-image separation). Furthermore, measured target angle noise on aircraft is typically Gaussian shaped with significant tails extending beyond the extremities of the target.

One effective method for reducing angle noise or glint is to use frequency agility. The authors discuss this approach in Appendix A of Ref. 1. However, this work would have been much more effective if they had taken advantage of the available literature such as the several papers on the subject by G. Ling (see listing under Frequency Agility and Diversity in Refs. 2 and 6). The authors essentially state that if the path distance between interfering reflectors is greater than the velocity of light ($c$) divided by the bandwidth (assuming $\lambda_0 = cV_0$, where $\lambda_0$ is the wavelength at center frequency $V_0$) the signals from the reflecting elements cannot interfer. This is later qualified by the authors, but as it stands it is a false statement and should not be made. The fact that the statement is incorrect is an important distinction between incoherent frequency agility and coherent use of the bandwidth for range resolution, for example. With the bandwidth used for range resolution, the elements can be resolved and interference prevented, as the reflectors separate further, while with frequency diversity they will continue to interfere.

In summary, I believe some useful assessment of the tracking or homing problems of Ref. 1 could have been made by the authors with a knowledge of the existing literature which is a result of many years of effort by many people. Without this background, the authors have made misleading interpretations and conclusions.

Also, the authors calculate the echo amplitude fluctuations but only speculate on the angle errors they cause. This results in a very weak paper when quantitative analysis rather than speculation is needed.

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## Reply by Authors to D.D. Howard


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MR. HOWARD'S criticism of the gap in the literature background of angle noise is quite proper. The authors should have presented such material, even though their conclusions do not involve such a type of aimpoint wander.

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The authors concede that, in principle, aimpoint wander may exist due to either intensity modulation or angle wander. In that connection, the criticism of the discussion of the observations as illustrated in Fig. 6 is justified, in that the spikes in the error signal record marked e, f, and g may truly be aimpoint excursions due to interference of echoes from the target, and that the echoes need not necessarily originate from the terrain just because the direction from which they appear to come falls off the target.

It is also true, however, that the error signals in the data presented in the paper are strictly products of angular error and signal intensity. It follows that the spikes in the error signal record may be due not to an aimpoint wander, in the sense of Mr. Howard's suggestion, but only to a sudden increase in echo intensity, as is our conclusion. The primary datum in support of this conclusion is that the spikes in the error signal record are always quickly damped. This is reasonably concluded to be due to the use of the AGC, if the cause of the spike is a sudden increase in signal intensity. There is no reason to expect such sudden damping if the spike is a true aimpoint wander.

We thank Mr. Howard for his helpful and thoughtful comments.

# Comment on "Optimizing the Gains of the Baro-Inertial Vertical Channel"

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## Introduction

IN the paper "Optimizing the Gains of the Baro-Inertial Vertical Channel" by Widnall and Sinha,[1] the selection of the three vertical channel gains was formulated as a stochastic optimal control problem, where the objective was to minimize the mean-square error of the indicated vertical velocity. A fifth-order error model was the basis for the analysis. An analytical expression was found for the mean-square vertical velocity error as a function of the assumed statistics of the sources of error and of the undetermined gains. A computer program was used to find the set of gain values that minimizes the mean-squared error. Sensitivity of the results to the statistical assumptions was explored. Analysis of the numerical results led to the discovery of approximate analytical formulas giving the optimal gains and pole locations as a function of the assumed statistics of the sources of error.

This note points out that the principal results of the paper could have been obtained without the use of a numerical minimum-finding computer program. This eliminates the usual concern associated with using such a program—namely, that the program may have only found a local minimum. It is shown that there is a third-order optimal estimator whose estimation error is governed by the same differential equation as that governing a third-order subsystem of the vertical channel. The results from optimal estimation theory therefore can be used for the vertical channel optimization problem. The poles of the optimal estimator (and the optimized vertical channel) are governed by a symmetric root characteristic equation. Having determined the optimal poles, the optimal gains may be calculated from the pole locations. This note also points out that the approximate analytical formulas for the optimal pole locations may be derived directly from the symmetric root characteristic equation.

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## Related Optimal Estimation Problem

The fifth-order error model used in the paper and in this note is

$$\delta\dot{h} = \delta v_z + u_1 \qquad \delta\dot{v}_z = c\delta h - \delta\hat{a} + \delta a + u_2 + w_{a1}$$
$$\delta\dot{\hat{a}} = -u_3 \qquad \delta\dot{a} = w_{a2} \qquad \delta\dot{b} = w_{b2} \tag{1}$$

where $\delta h$ is the error in indicated altitude, $\delta v_z$ is the error in indicated vertical velocity, $\delta\hat{a}$ is the computed vertical acceleration error, $\delta a$ is the slowly varying acceleration error, and $\delta b$ is the slowly varying error in altitude indicated by the barometric altimeter. $w_{a1}$, $w_{a2}$, $w_{b2}$ are white noises of spectral density $Q_{a1}$, $Q_{a2}$, $Q_{b2}$ which provide the short correlation time acceleration error, the acceleration error random walk, and the altimeter error random walk. $u_1$, $u_2$, $u_3$ are control variables. The constant $c$ is the gravity gradient constant whose value near the surface of the earth is $c = 2g/R = 3.07 \times 10^{-6}\ \text{s}^{-2}$. The measurement, from which are derived the corrections to the vertical channel variables, is the difference between the indicated altitude and the altitude indicated by the barometric altimeter. In terms of the errors in these variables, the measurement is

$$y = \delta h - \delta b - w_{b1} \tag{2}$$

where $w_{b1}$ is the white noise of the spectral density $Q_{b1}$ modeling short correlation time altimeter error. In the baro-inertial vertical channel, the control variables are simply

$$u_1 = -k_1 y \qquad u_2 = -k_2 y \qquad u_3 = -k_3 y \tag{3}$$

It can be shown the fifth-order state vector is not completely observable through the measurement $y$. A bias acceleration error $\delta a$ offset by a bias computed acceleration error $\delta\hat{a}$ cannot be observed through the measurement $y$, and a bias altimeter error $\delta b$ offset by a bias indicated altitude $\delta h$ cannot be observed through the measurement $y$. A third-order subsystem that is completely observable has these state variables:

$$x_1 = \delta h - \delta b \qquad x_2 = \delta v_z \qquad x_3 = -\delta\hat{a} + \delta a + c\delta b \tag{4}$$

This subsystem and the measurement are governed by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ c & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & c \end{bmatrix} \begin{bmatrix} w_{a1} \\ w_{a2} \\ w_{b2} \end{bmatrix} \tag{5}$$

$$y = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - w_{b1} \tag{6}$$